PHY2022

UNIVERSITY OF EXETER

PHYSICS

MID TERM TEST

Student No.:		Candidate No.:	
Personal tutor:			
Degree programme:			

Specimen Paper

(Time allowed: 30 minutes)

Attempt **ALL questions**. All essential working should be shown. The mark for each question or part of a question is shown in square brackets.

A correct answer to a multiple choice question gains the marks indicated, the "Don't know" answer 'x' gets zero. An incorrect answer is penalised by the deduction of one mark unless sufficient working is shown to justify the choice, in which case it will get zero.

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1. For a normalized wave function $\psi(x)$, the quantity $\int_{-\infty}^{\infty} |\psi(x)|^2\, dx$ is:

(a) a probability density; $|\psi(x)|^2$

(b) a probability amplitude; $\psi(x)$

(c) 1;

(x) Don't know. Answer: ______ **[2 marks]**

Turn Over

2. The expectation value of the observable A represented by the operator $\hat{A}$, in the state whose normalized wave function is $\psi(x)$, is

(a) $\int_{-\infty}^{\infty} \psi^*(x)\psi(x)dx$;

(b) $\int_{-\infty}^{\infty} \psi^*(x)\hat{A}\psi(x)dx$;

(c) $\int_{-\infty}^{\infty} \hat{A}|\psi(x)|^2 dx$;

(x) Don't know. Answer: ______ **[2 marks]**

3. The wave function $\Psi(x,t) = \Psi_0 \exp[i(kx + \omega t)]$ represents:

$\Psi(x,t) = \Psi_0 \exp[i(kx - \omega t)]$

(a) A free particle having energy $\hbar\omega$ travelling in the positive x direction;

(b) A free particle having energy $\hbar\omega$ travelling in the negative x direction;

(c) A particle having energy $\hbar\omega$ bound within an infinite square well;

(x) Don't know. Answer: ______ **[2 marks]**

e.g. anti-symm:

$\Psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$

Turn Over

4. Which of the following four wave functions is **not** normalizable?

$$\psi_1(x) = A\exp(-|x|), \text{ for all } x;$$
$$\psi_2(x) = \begin{cases} \cos(\pi x/2L), \text{ for } -L \le x \le L \\ 0, \text{ elsewhere;} \end{cases}$$
$$\psi_3(x) = A\exp(-x), \text{ for all } x;$$
$$\psi_4(x) = \begin{cases} A\exp(-x), \text{ for } x \ge 0 \\ 0, \text{ elsewhere.} \end{cases}$$

(a) $\psi_1(x)$;

(b) $\psi_2(x)$;

(c) $\psi_3(x)$;

(d) $\psi_4(x)$;

(x) Don't know. Answer: ______ **[3 marks]**

5. When a measurement is made of an observable A on a state Ψ which is not an eigenstate of the operator $\hat{A}$ which of the following statements is **not** true:

$\underline{\underline{\hat{A}\psi \neq a\psi}}$

(a) immediately after the measurement the system will be in the state described by Ψ; false

(b) the mean of several such measurements is given by the expectation value of A in the state Ψ;

(c) the result of the experiment is uncertain;

(d) Don't know. Answer: ______ **[2 marks]**

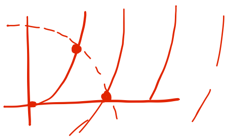
Turn Over

6. In comparison with a one-dimensional infinite square well, a one-dimensional finite square well of the same width,

 fin.

 inf.

Remember FINITE SQUARE WELL:



(a) has a ground state whose energy is higher than that of the infinite well and is always bound;

(b) has a ground state whose energy is lower than that of the infinite well and is always bound;

(c) has a ground state whose energy is higher than that of the infinite well and may not be bound;

(d) has a ground state whose energy is lower than that of the infinite well and may not be bound;

(x) Don't know. Answer: ______ **[3 marks]**

7. A normalized wave function is given by

$$
\begin{aligned}
\psi(x) &= 0 && \text{for } x < -L \\
\psi(x) &= A(x+L) && \text{for } -L \le x \le 0 \\
\psi(x) &= A(L-x) && \text{for } 0 < x \le L \\
\psi(x) &= 0 && \text{for } x > L.
\end{aligned}
$$

The normalization constant A may be equal to:

$$A^2 \int_{-L}^{0} (x+L)^2 \, dx = A^2 \int_{-L}^{0} (x^2 + 2xL + L^2) \, dx$$

$$= A^2 \left[\frac{x^3}{3} + x^2 L + x L^2 \right]_{-L}^{0}$$

$$= -A^2 \left[-\frac{L^3}{3} + L^3 - L^3 \right] = A^2 \frac{L^3}{3}$$

$$A^2 \int_{0}^{L} (x-L)^2 \, dx = A^2 \left[\frac{x^3}{3} - x^2 L + x L^2 \right]_{0}^{L}$$

$$= A^2 \left[\frac{L^3}{3} - L^3 + L^3 \right] = A^2 \frac{L^3}{3}$$

Turn Over

$$\int_{-\infty}^{+\infty} |\psi(x)|^2 \, dx = A^2 \frac{2}{3} L^3$$

$$A^2 \frac{2}{3} L^3 = 1 \qquad A = \sqrt{\frac{3}{2L^3}}$$

(a) $\dfrac{3}{L^3}$;

(b) $\dfrac{3}{2L^3}$;

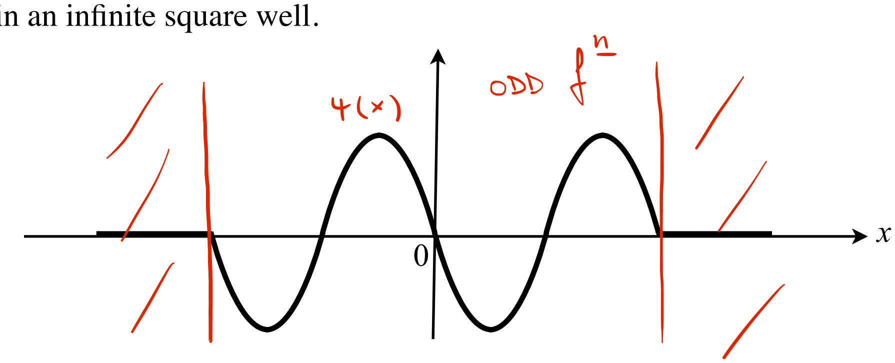
(c) $\sqrt{\dfrac{1}{L}}$;

(d) $\sqrt{\dfrac{3}{L^3}}$;

(e) $\sqrt{\dfrac{3}{2L^3}}$.

(x) Don't know. Answer: ______ **[4 marks]**

8. The figure below shows the wave function corresponding to an energy eigenstate for a particle in an infinite square well.



Which of the following statements is correct:

(a) The wave function is an even function so the probability density will be an even function;

(b) The wave function is an even function so the probability density will be an odd function;

(c) The wave function is an odd function so the probability density will be an odd function;

(d) The wave function is an odd function so the probability density will be an even function; $\psi(x) = \sin x \qquad \psi(x)^2 = \sin^2(x)$

(x) Don't know. Answer: ______ **[3 marks]**

Turn Over

9. Consider a particle confined in one dimension within the infinite square well potential

$$V(x) = \begin{cases} 0 & (-a \le x \le a) \\ \infty & \text{elsewhere.} \end{cases}$$

The normalized ground-state wave function is $u(x) = \frac{1}{\sqrt{a}} \cos\left(\frac{\pi x}{2a}\right)$.

The expectation value of x^2, $\langle x^2 \rangle$, for a particle in the ground state is:

$$\int_{-a}^{a} u^*(x)\, x^2\, u(x)\, dx = \frac{1}{a} \int_{-a}^{a} x^2 \cos^2\left(\frac{\pi x}{2a}\right) dx$$

$$= \frac{1}{a} \int_{-a}^{a} x^2 \left(\frac{1 + \cos\left(\frac{\pi x}{a}\right)}{2} \right) dx$$

by parts

$$= \frac{1}{2a} \int_{-a}^{a} x^2\, dx + \frac{1}{2a} \int_{-a}^{a} x^2 \cos\left(\frac{\pi x}{a}\right) dx$$

$$= \frac{1}{2a} \left. \frac{x^3}{3} \right|_{-a}^{a} + \frac{1}{2a} \left\{ \underbrace{\left. x^2 \frac{a \sin\left(\frac{\pi x}{a}\right)}{\pi} \right|_{-a}^{a}}_{0} - \int_{-a}^{a} 2x \frac{a \sin\left(\frac{\pi x}{a}\right)}{\pi} dx \right\}$$

(a) $\frac{a^2}{3}$;

(b) $\frac{a^2}{2}$;

(c) $\frac{a^2}{3} - \frac{2a^2}{\pi^2}$;

(d) $\frac{a^2}{3} - \frac{a^2}{\pi^2}$;

(e) a^2.

$$= \frac{a^2}{3} - \frac{1}{\pi} \int_{-a}^{a} x \sin\left(\frac{\pi x}{a}\right) dx$$

$$= \frac{a^2}{3} - \frac{1}{\pi} \left\{ \left. -x \frac{a}{\pi} \cos\left(\frac{\pi x}{a}\right) \right|_{-a}^{+a} + \right.$$

$$\left. - \int_{-a}^{a} -\frac{a}{\pi} \underbrace{\cos\left(\frac{\pi x}{a}\right) dx}_{0} \right\}$$

(x) Don't know. Answer: ______ **[4 marks]**

END OF PAPER

$$= \frac{a^2}{3} + \frac{a}{\pi^2} \left. x \cos\left(\frac{\pi x}{a}\right) \right|_{-a}^{a}$$

$$= \frac{a^2}{3} + \frac{a}{\pi^2} \left[a \cos(\pi) + a \cos(\pi) \right] = \frac{a^2}{3} - 2 \frac{a^2}{\pi^2}$$

Answers

1. c
2. b
3. b
4. c
5. a
6. b
7. e
8. d
9. c