MID - TERM SOLUTIONS

1. For a normalized wave function $\psi(x)$, the quantity $|\psi(x)|^2$ is:

(a) a probability density;
(b) a probability amplitude;
(c) 1;
(x) Don't know.

2. The probability of finding a particle, whose normalized wave function is $\psi(x)$, in the region between x and $x + dx$ is:

(a) $\psi(x)\, dx$;
(b) $(\psi(x))^2\, dx$;
(c) $|\psi(x)|^2\, dx$;
(x) Don't know.

3. The operator for the x-component of momentum is:

(a) $\hbar \frac{\partial}{\partial x}$;
(b) $i\hbar \frac{\partial}{\partial x}$;
(c) $\frac{\hbar}{i} \frac{\partial}{\partial x}$;
(x) Don't know.

$$\hat{p}_x = \frac{\hbar}{i} \frac{\partial}{\partial x} = \frac{i}{i} \frac{\hbar}{i} \frac{\partial}{\partial x}$$

$$= -i\hbar \frac{\partial}{\partial x}$$

4. Which of the following four wave functions is normalizable?

$$\begin{aligned} \psi_1(x) &= A\exp(ikx) && \text{for all } x; \\ \psi_2(x) &= A && \text{for } -L \le x \le L, \text{ and} \\ &= 0 && \text{elsewhere;} \\ \psi_3(x) &= A\exp(-kx) && \text{for all } x; \\ \psi_4(x) &= Ax && \text{for } x \le -L, \text{ and} \\ &= 0 && \text{elsewhere;} \end{aligned}$$

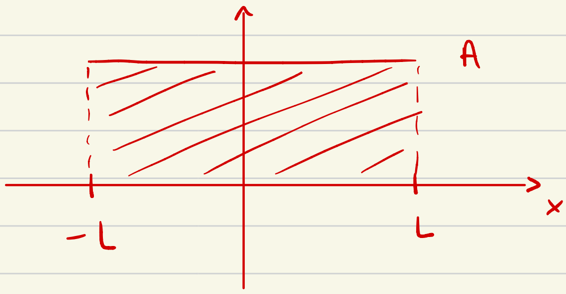
(a) $\psi_1(x)$;

(b) $\psi_2(x)$;

(c) $\psi_3(x)$;

(d) $\psi_4(x)$;

(x) Don't know.



5. In quantum mechanics, a dynamical variable is described by an operator whose expectation value is:

(a) the most probable value of the quantity;

(b) the mean value of the quantity;

(c) the standard deviation of the quantity;

(x) Don't know.

6. A one-dimensional finite square well of width a consists of a region of zero potential energy inside the well and barriers of height V_0. As a is reduced,

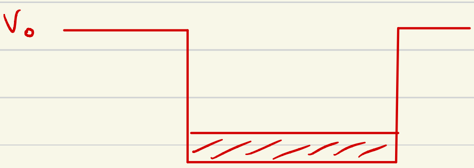
(a) the energies of all the bound states increase, and below a certain value of a no bound states remain;

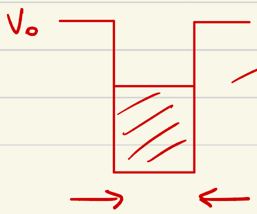
→ always a symm. bound state

(b) the energies of all the bound states decrease, and below a certain value of a no bound states remain;

(c) the energies of all the bound states decrease, but for all values of $a > 0$ at least one bound state remains;

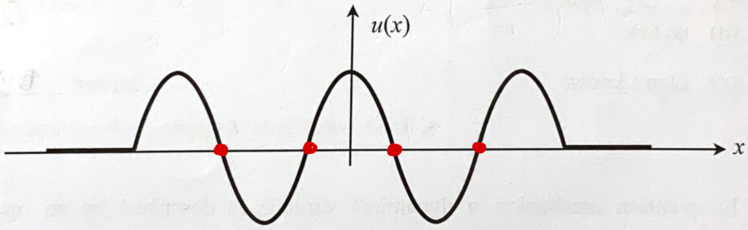
(d) the energies of all the bound states increase, but for all values of $a > 0$ at least one bound state remains.





energy increases

7. The figure below shows the wave function corresponding to an energy eigenstate for a particle confined to an infinite square well.

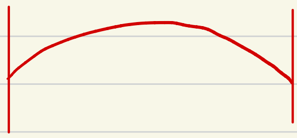


The quantum number n associated with this eigenstate (taking the ground state as $n = 1$) is:

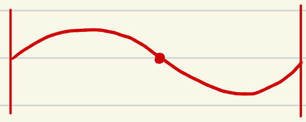
(a) 6;
(b) 5; ✓
(c) 4;
(d) 3;
(x) Don't know.

Start from

$n = 1$
$\emptyset$ nodes



$n = 2$
1 node



⋮

$n = 5$
4 nodes

8. A normalized wave function is given by

$$\psi(x) = A\,x \exp(-x/L) \qquad \text{for } x \geq 0, \text{ and}$$
$$= 0 \qquad \text{for } x < 0.$$

The normalization constant A may be equal to:

$$\int |\psi(x)|^2 \, dx = 1$$

$$A^2 \int_0^\infty x^2 e^{-\frac{2x}{L}} \, dx = 1$$

(by parts)

0 - 0

$$= A^2 \left[\cancel{x^2 \left(-\frac{L}{2}\right) e^{-\frac{2x}{L}} \Big|_0^\infty} - \int_0^\infty 2x \left(-\frac{L}{2}\right) e^{-\frac{2x}{L}} \, dx \right]$$

$$= L A^2 \int_0^\infty x \, e^{-\frac{2x}{L}} \, dx$$

0 - 0

$$= L A^2 \left[\cancel{x \left(-\frac{L}{2}\right) e^{-\frac{2x}{L}} \Big|_0^\infty} - \int_0^\infty \left(-\frac{L}{2}\right) e^{-\frac{2x}{L}} \, dx \right]$$

$$= LA^2 \frac{L}{2}\left(-\frac{L}{2}\right) e^{-\frac{2x}{L}} \Big|_0^\infty$$

$$= -A^2 \frac{L^3}{4} (0 - 1)$$

$$= A^2 \frac{L^3}{4} = 1 \quad \Rightarrow \quad A^2 = \frac{4}{L^3}$$

$$\boxed{A = \frac{2}{L^{3/2}}}$$

9. The normalized ground-state wave function of a particle confined in one dimension within the infinite square well potential

$$V(x) = 0 \qquad \text{for } 0 \le x \le a \text{, and}$$
$$= \infty \qquad \text{elsewhere,}$$

is $u(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$.

The expectation value of the square of the x-component of the momentum, $\langle p_x^2 \rangle$, for the particle in the ground state is:

$$\langle p_x^2 \rangle = \int u^*(x)\, p_x^2\, u(x)\, dx \qquad \hat{p}_x = -i\hbar \partial_x$$

$$p_x^2 = -\hbar^2 \partial_x^2$$

$$\langle p_x^2 \rangle = \int_0^a \frac{2}{a} \sin\left(\frac{\pi x}{a}\right) -\hbar^2 \partial_x^2 \sin\left(\frac{\pi x}{a}\right) dx$$

$$= -\frac{2\hbar^2}{a} \int_0^a \sin\left(\frac{\pi x}{a}\right)\left(-\frac{\pi^2}{a^2}\right) \sin\left(\frac{\pi x}{a}\right) dx$$

$$= \frac{2\hbar^2 \pi^2}{a^3} \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx$$

$$= \frac{2\hbar^2\pi^2}{a^3} \int_0^a \frac{1 - \cos\left(\frac{2\pi x}{a}\right)}{2} \, dx$$

$$= \frac{2\hbar^2\pi^2}{a^3} \left[\int_0^a \frac{1}{2} \, dx - \frac{1}{2} \int_0^a \cos\left(\frac{2\pi x}{a}\right) dx \right]$$

$0 - 0$

$$= \frac{2\hbar^2\pi^2}{a^3} \left[\frac{a}{2} - \frac{1}{2} \frac{a}{2\pi} \sin\left(\frac{2\pi x}{a}\right) \Big|_0^a \right]$$

$$= \frac{2\hbar^2\pi^2}{a^3} \frac{a}{2}$$

$$\boxed{\langle p_x^2 \rangle = \frac{\hbar^2\pi^2}{a^2}}$$